

# $(g - 2)_\mu$ and Supersymmetry

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Zaragoza, 05/2004

based on collaboration with  
*D. Stöckinger and G. Weiglein*

1. Introduction
2. The anomalous magnetic moment of the Muon
3. Calculation of MSSM two-loop corrections
4. MSSM two-loop results (incl. very recent results)
5. Conclusions

## 1. Introduction

**Q:** Which Lagrangian describes the world?

**Q':** What describes the world better: SM or MSSM ?

**A:** Two possible ways:

- Search for new SUSY particles

new SUSY particles found

$\Updownarrow$

SM ruled out

Problem:

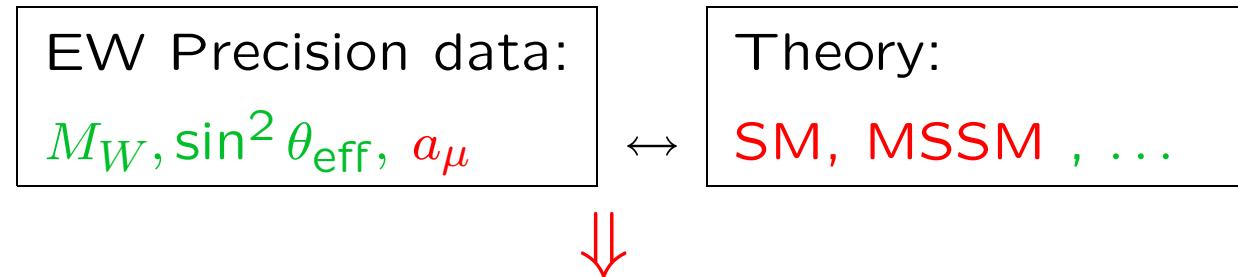
SUSY particles are too heavy for todays  
colliders, only upper limits of  $\mathcal{O}(100 \text{ GeV})$ .

- waiting for Tevatron (2005 ... ?)
- waiting for LHC (2007 ... ?)

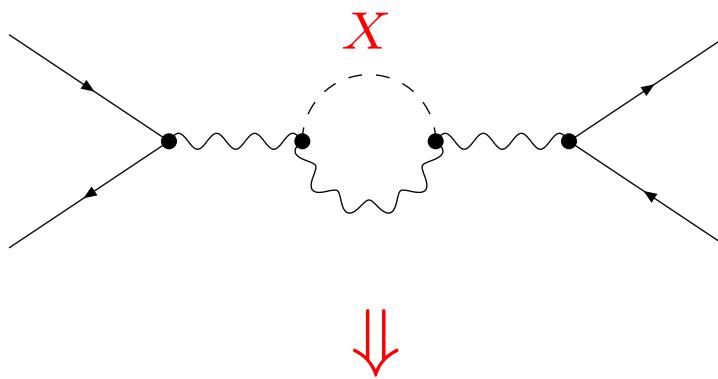
- Search for indirect effects of SUSY  
via Precision Observables

## Precision Observables (POs):

Comparison of electro-weak precision observables with theory:



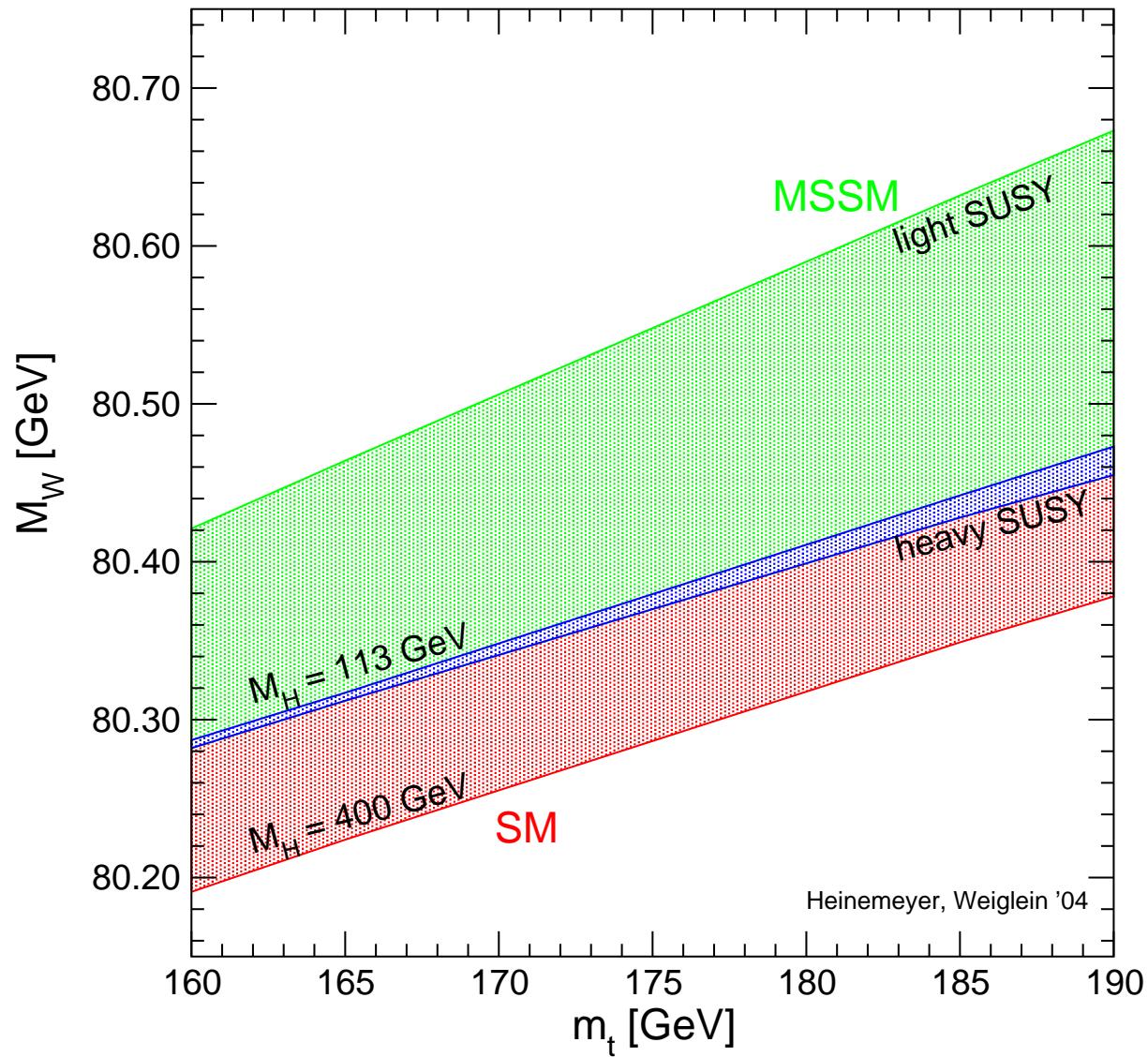
Test of theory at quantum level: **Sensitivity to loop corrections**



**Very high accuracy of measurements and theoretical predictions needed**

- Which model fits better?
- Does the prediction of a model contradict the experimental data?

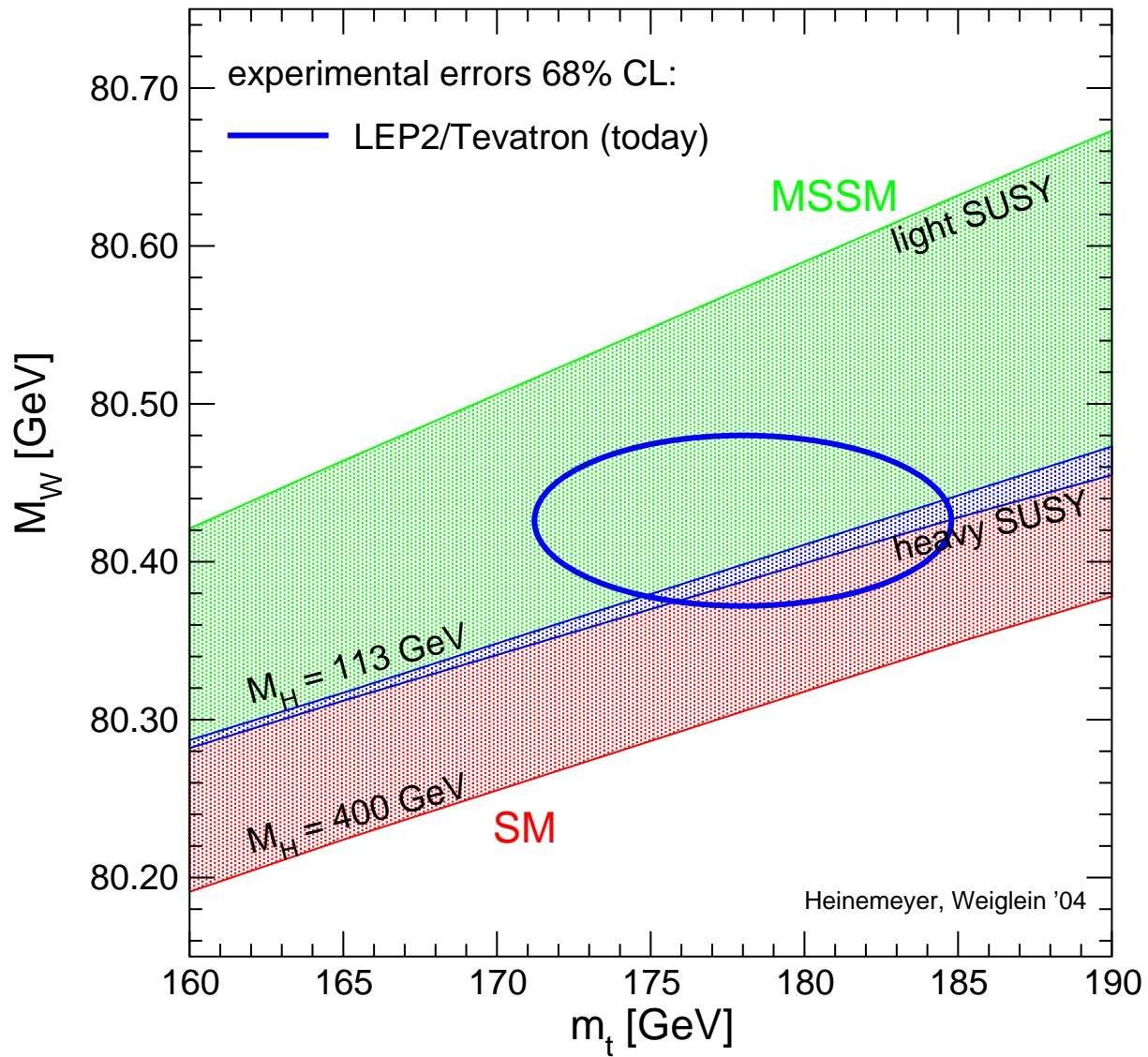
## Example: Prediction for $M_W$ in the SM and the MSSM :



**MSSM uncertainty:**  
unknown masses  
of SUSY particles

**SM uncertainty:**  
unknown Higgs mass

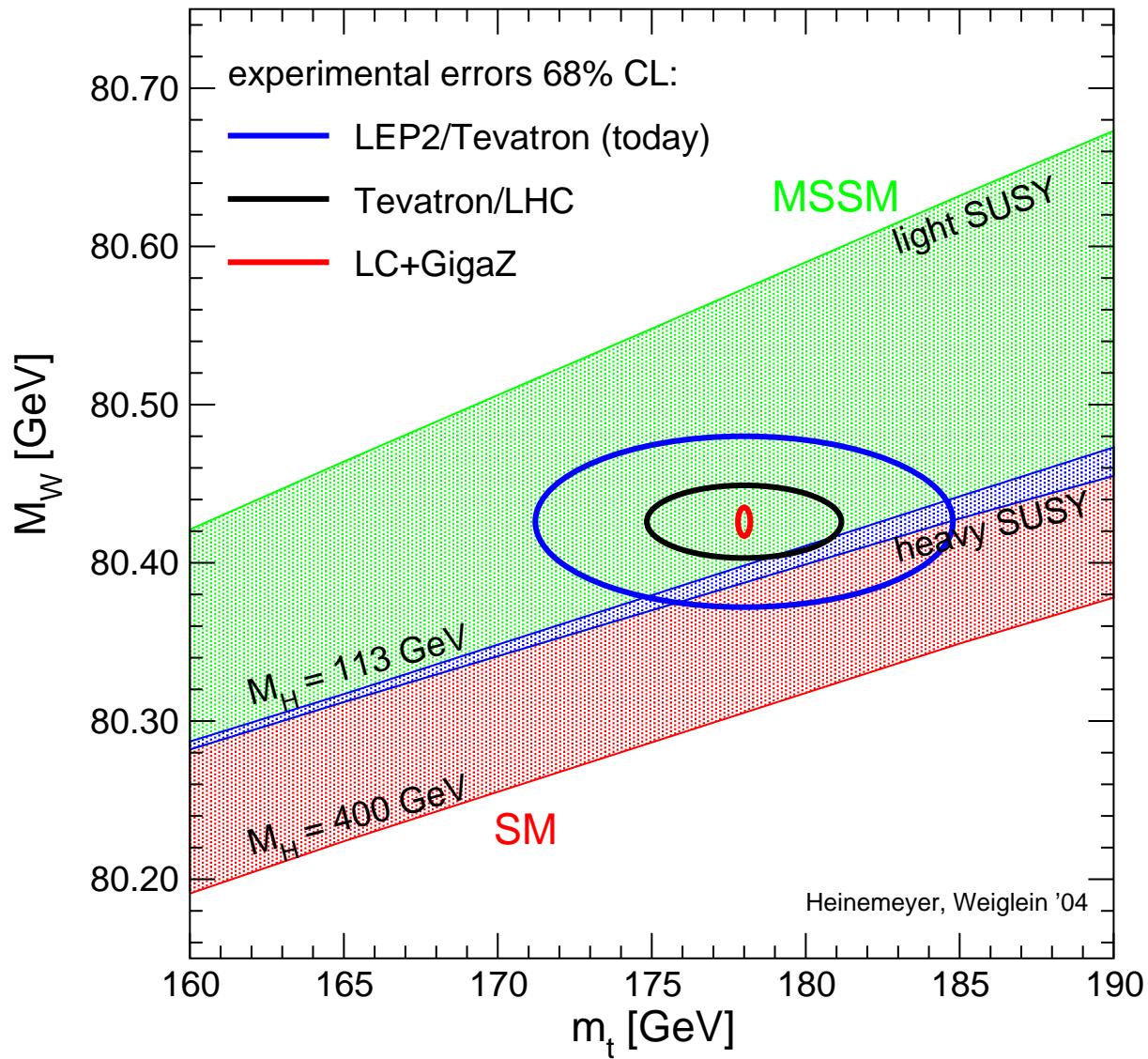
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# The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles

$[u, d, c, s, t, b]_{L,R}$	$[e, \mu, \tau]_{L,R}$	$[\nu_{e,\mu,\tau}]_L$	Spin $\frac{1}{2}$
$[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}]_{L,R}$	$[\tilde{e}, \tilde{\mu}, \tilde{\tau}]_{L,R}$	$[\tilde{\nu}_{e,\mu,\tau}]_L$	Spin 0
$g$	$\underbrace{W^\pm, \textcolor{red}{H}^\pm}_{\textcolor{blue}{W^\pm, H^\pm}}$	$\underbrace{\gamma, Z, \textcolor{red}{H}_1^0, H_2^0}_{\textcolor{blue}{\gamma, Z, H_1^0, H_2^0}}$	Spin 1 / Spin 0
$\tilde{g}$	$\tilde{\chi}_{1,2}^\pm$	$\tilde{\chi}_{1,2,3,4}^0$	Spin $\frac{1}{2}$

Enlarged Higgs sector: Two Higgs doublets

Problem in the MSSM: many scales

## Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}}_{\text{gauge couplings, in contrast to SM}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}_{\text{gauge couplings, in contrast to SM}} |H_1 \bar{H}_2|^2$$

physical states:  $h^0, H^0, A^0, H^\pm$

Goldstone bosons:  $G^0, G^\pm$

Input parameters:

$$\tan \beta = \frac{v_2}{v_1}, \quad M_A^2 = -m_{12}^2 (\tan \beta + \cot \beta)$$

→ Prediction for  $m_h$ :

MSSM tree-level bound:  $m_h < M_Z$ , excluded by LEP Higgs searches

Large radiative corrections:

Dominant one-loop corrections:  $\sim G_\mu m_t^4 \ln \left( \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right)$

Measurement of  $m_h$ , Higgs couplings ⇒ test of the theory

LHC:  $\Delta m_h \approx 0.2$  GeV, LC:  $\Delta m_h \approx 0.05$  GeV

⇒  $m_h$  will be (the best?) electroweak precision observable

Present status of  $m_h$  prediction in the MSSM:

Feynman-diagrammatic result:

complete one-loop, leading + subleading two-loop

*FeynHiggs2.1* ([www.feynhiggs.de](http://www.feynhiggs.de))

[S. H., W. Hollik, G. Weiglein '98, '00, '02]

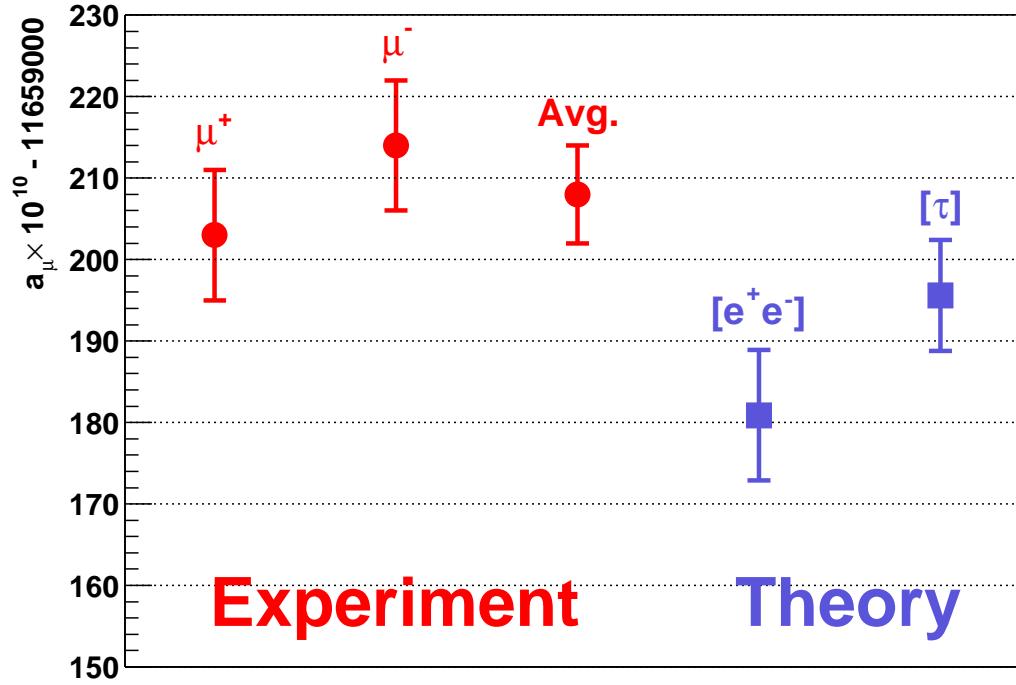
[T. Hahn, S. H., W. Hollik, G. Weiglein '04]

⇒ used for later Higgs evaluation

## 2. The anomalous magnetic moment of the muon: $a_\mu = (g - 2)_\mu / 2$

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Overview about the experimental and SM (theory) result:  
[*g-2 Collaboration, hep-ex/0401008*]

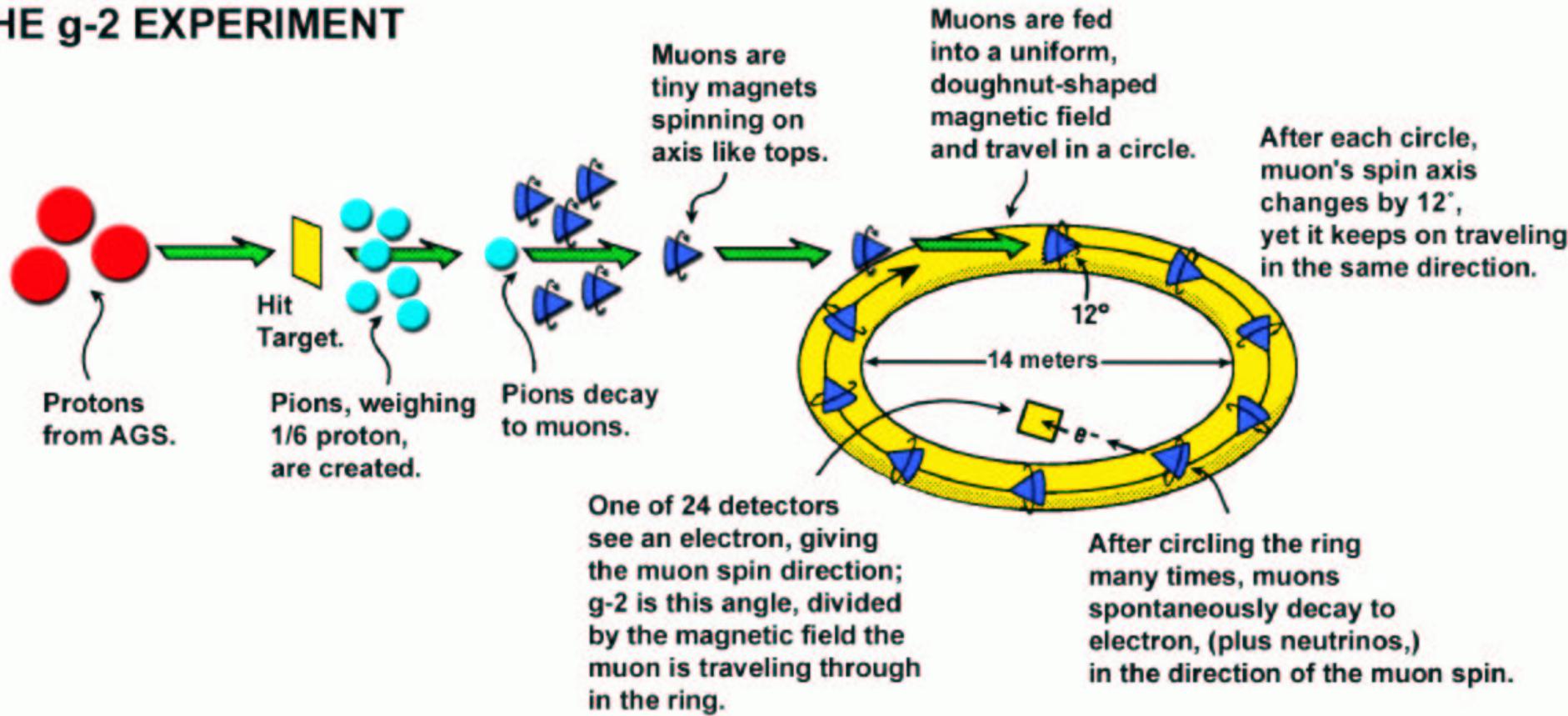


$$\text{Experiment : } a_\mu^{\text{exp}} = (11659208 \pm 6) \times 10^{-10}$$

$$\text{SM Theory : } a_\mu^{\text{theo}} \approx (11659181 \pm 8) \times 10^{-10}$$

# The $(g - 2)_\mu$ experiment:

## LIFE OF A MUON: THE g-2 EXPERIMENT



Coupling of muon to magnetic field :  $\mu - \mu - \gamma$  coupling

$$\bar{u}(p') \left[ \gamma^\mu F_1(q^2) + \frac{i}{2m_\mu} \sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p) A_\mu \quad F_2(0) = (g - 2)_\mu$$

## The SM theory evaluation:

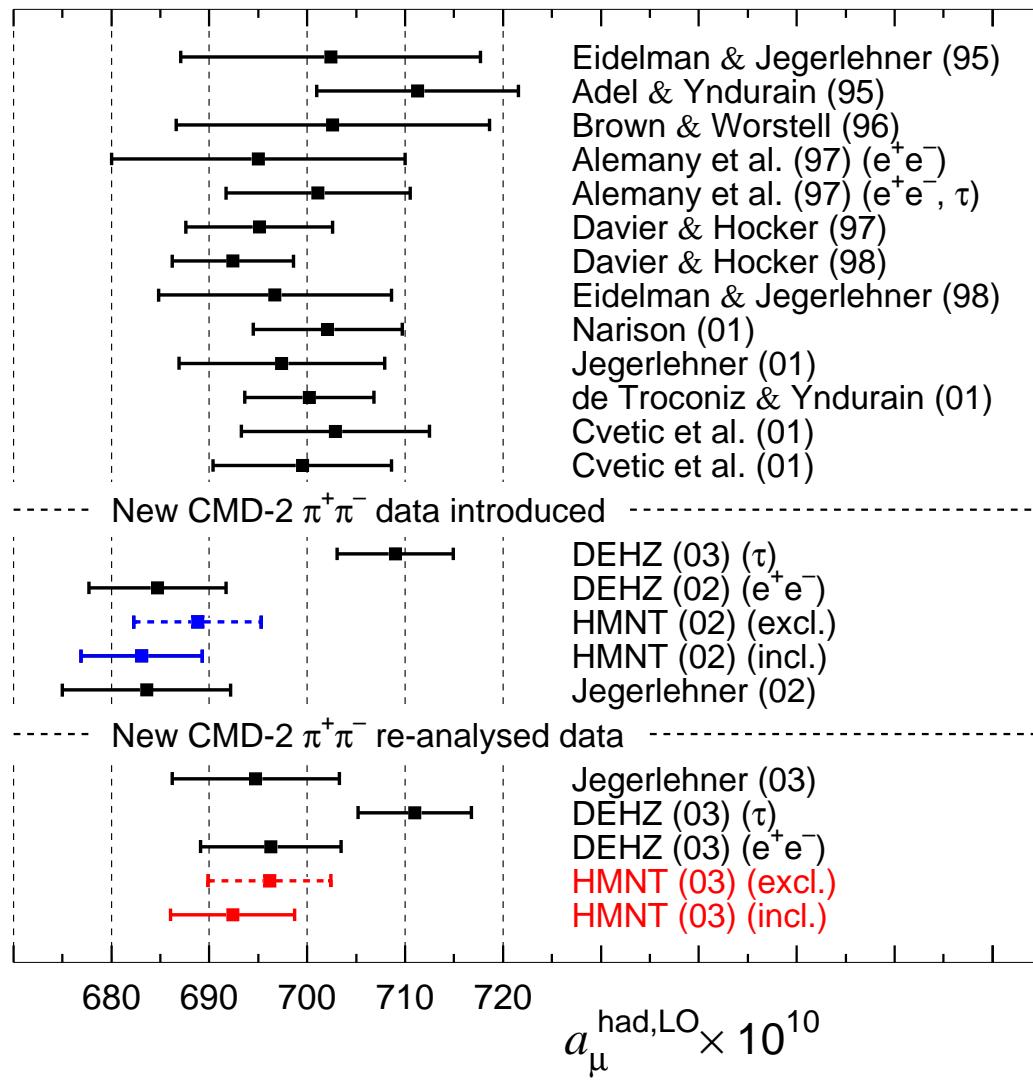
Source	contr. to $a_\mu [10^{-10}]$		
LO hadr.	$696.3 \pm 7.2$	$(e^+e^-)$	[Davier, Eidelman, Höcker, Zhang '03]
	$711.0 \pm 5.8$	$(\tau)$	[Davier, Eidelman, Höcker, Zhang '03]
	$694.8 \pm 8.6$	$(e^+e^-)$	[Ghozzi, Jegerlehner '03]
	$691.7 \pm 6.1$	$(e^+e^-)$	[Hagiwara, Martin, Nomura, Teubner '03]
LBL	$8 \pm 4$		[Knecht, Nyffeler '02]
	$13.6 \pm 2.5$	tbc	[Melnikov, Vainshtein '03]
EW 1L	19		
EW 2L	-4		[Czarnecki, Krause, Marciano '98]
exp. res.	6		

difference of  $\tau$  based and  $e^+e^-$  based LO hadr. is about  $2 - 3\sigma$

“Isospin breaking effects” in  $\tau$  based evaluation of LO hadr. not properly under control ? [Ghozzi, Jegerlehner '03]

⇒ concentrate on  $e^+e^-$  data

# The $a_\mu^{\text{had},\text{LO}}$ history:



## Deviation of $a_\mu^{\text{exp}}$ and $a_\mu^{\text{theo}}$ ( $e^+e^-$ ):

$$a_\mu^{\text{exp}} - a_\mu^{\text{theo}} \left[10^{-10}\right] \quad [\sigma]$$

$27 \pm 10.0$	2.7	[Davier, Eidelman, Höcker, Zhang '03]
$28 \pm 11.1$	2.5	[Ghozzi, Jegerlehner '03]
$32 \pm 9.5$	3.4	[Hagiwara, Martin, Nomura, Teubner '03]
$26 \pm 8.9$	3.0	including [Melnikov, Vainshtein '03]

## SUSY corrections at 1L:

$\rightarrow T$

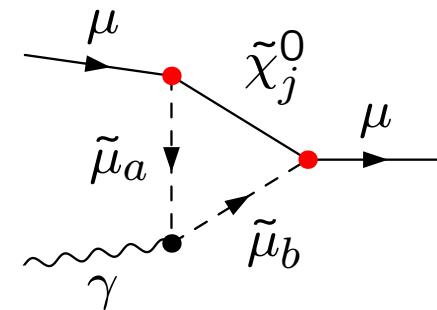
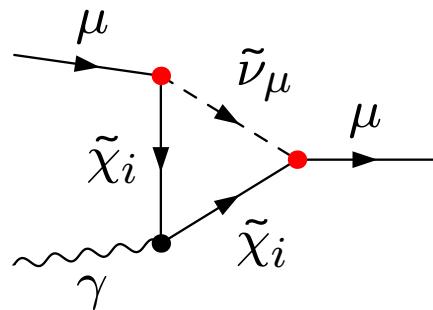
$$a_\mu^{\text{SUSY}} \approx 13 \times 10^{-10} \left( \frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 \tan \beta \text{ sign}(\mu)$$

$M_{\text{SUSY}}$ : generic SUSY mass scale

## SUSY corrections at 2L: ??

( $\rightarrow$  our calculation)

## Feynman diagrams for MSSM 1L corrections:



- Diagrams with chargino/sneutrino exchange
- Diagrams with neutralino/smuon exchange

Enhancement factor as compared to SM:

$$\mu - \tilde{\chi}_i^{\pm} - \tilde{\nu}_{\mu} : \sim m_{\mu} \tan \beta$$

$$\mu - \tilde{\chi}_j^0 - \tilde{\mu}_a : \sim m_{\mu} \tan \beta$$

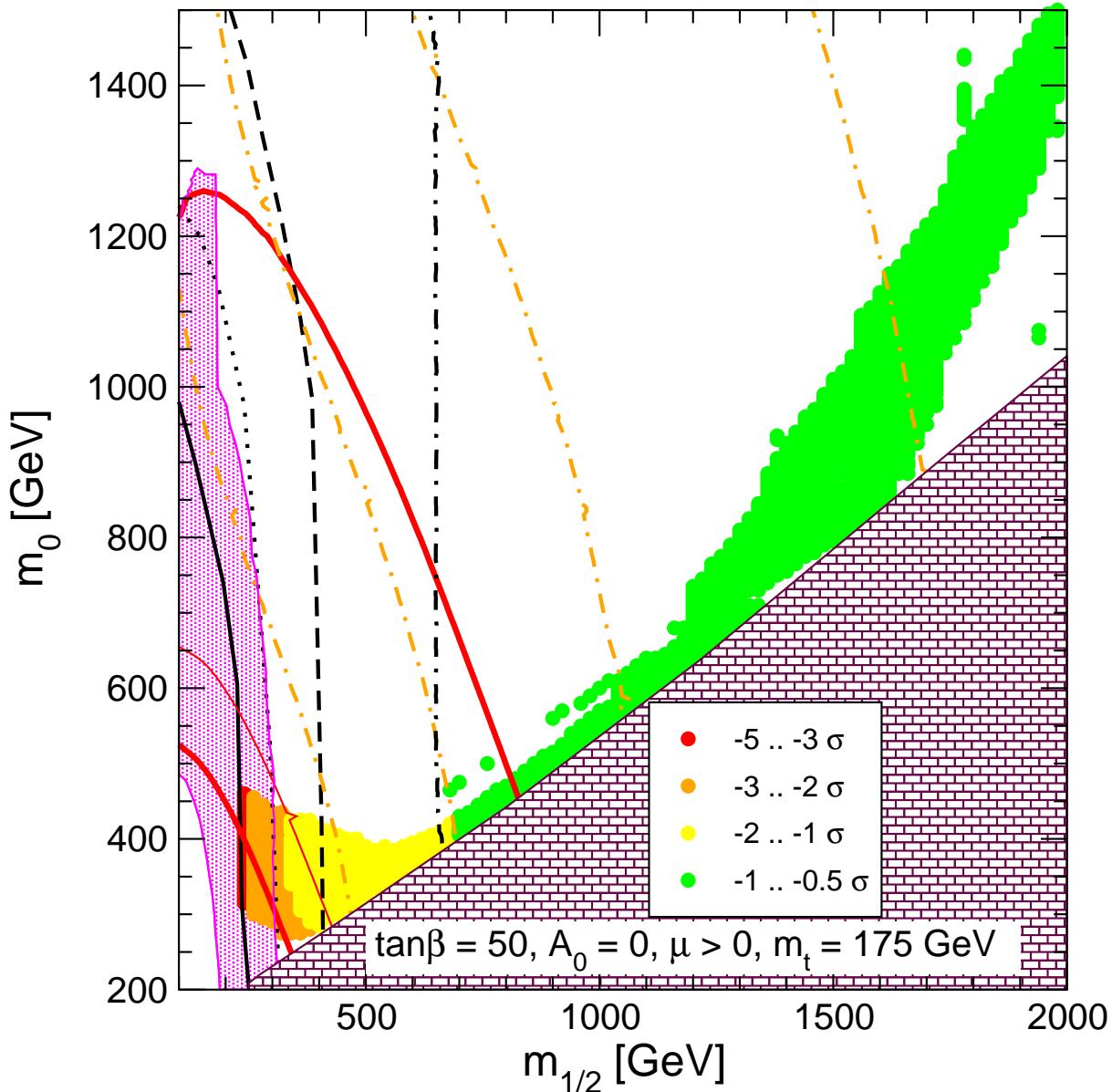
$$\text{SM, EW 1L: } \frac{\alpha}{\pi} \frac{m_{\mu}^2}{M_W^2}$$

$$\text{MSSM, 1L: } \frac{\alpha}{\pi} \frac{m_{\mu}^2}{M_{\text{SUSY}}^2} \times \tan \beta$$

SUSY could easily explain the “discrepancy”  
 $a_{\mu}$  can provide bounds on SUSY parameters

$\rightarrow T$

## Example for bounds on SUSY parameters: mSUGRA



mSUGRA:

$$\tan\beta = 50, A_0 = 0, \mu > 0$$

$\text{BR}(h \rightarrow WW^*)$ , MSSM/SM

[J.Ellis, S.H., K.Olive, G.Weiglein  
'02]

$$e^+e^-: \delta a_\mu = (33.9 \pm 11.2)$$

$$\tau: \quad \delta a_\mu = (16.7 \pm 10.7)$$

Already known:

QED corrections to  $a_{\mu}^{\text{SUSY}}(1L)$ :  $\sim -7\%$

[*G. Degrassi, G. Giudice '98*]

Already known:

Approximation of leading terms can be very large, up to  $\sim 20 \times 10^{-10}$

[*C. Chen, C. Geng '01*] , [*A. Arhrib, S. Baek '01*]

However: results disagree by a factor of 4!

Questions for numerical evaluation:

**Q1:** How large are the complete contributions?

**Q2:** What happens if experimental constraints are taken into account?

## Our goal:

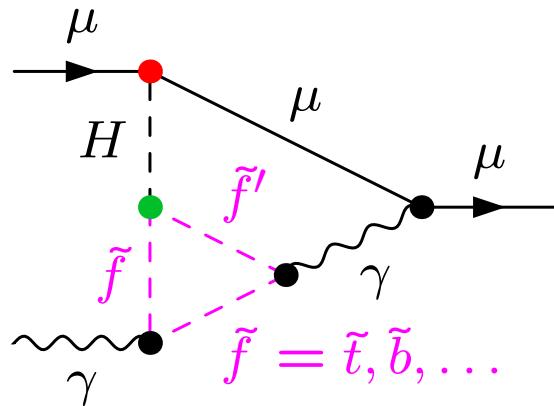
all corrections to SM/THDM diagrams with a closed fermion or scalar fermion loop

→ T

## Possible enhancement by:

- $t, b, \tau$  Yukawa couplings
- large  $\mu$  and/or  $A_f$  and/or  $\tan\beta$  (in couplings)
- small  $m_{\tilde{t}}, m_{\tilde{b}}, m_{\tilde{\tau}}$

Example: Barr-Zee type diagram:



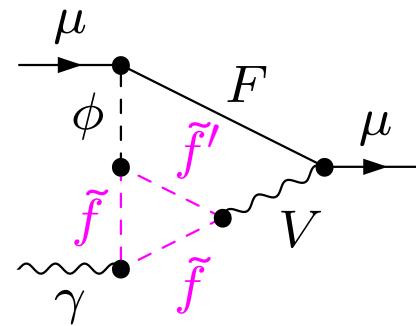
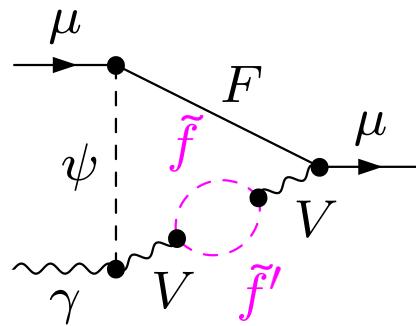
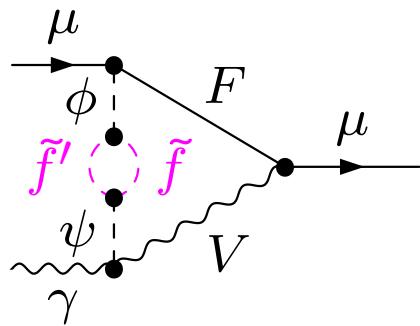
$$\begin{aligned} &\rightarrow m_\mu \tan\beta \\ &\rightarrow \mu m_t, m_b A_b \tan\beta \end{aligned}$$

⇒ Enhancement:

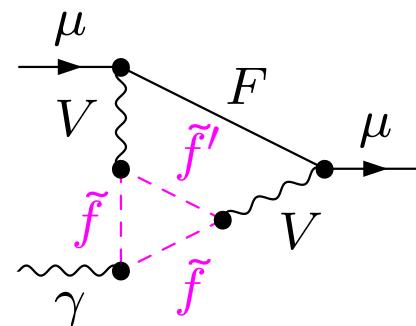
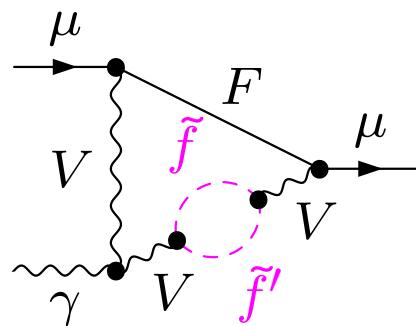
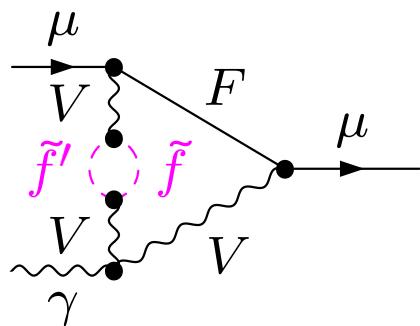
$$a_\mu \sim \tan\beta \frac{\mu m_t}{M_H m_{\tilde{t}}}$$

$$a_\mu \sim \tan^2\beta \frac{m_b A_b}{M_H m_{\tilde{b}}}$$

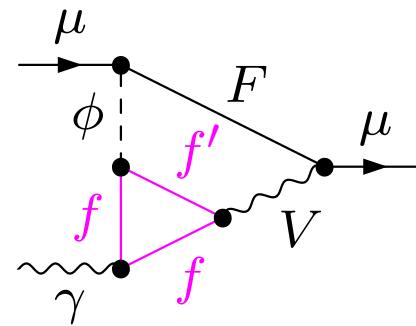
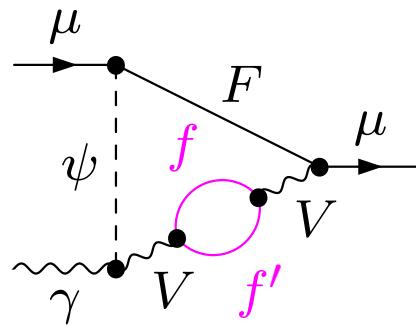
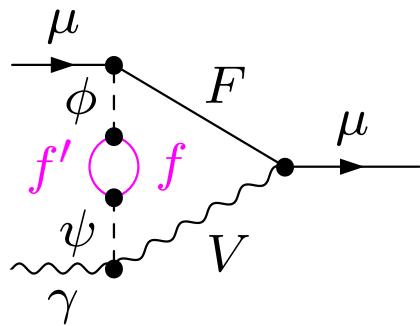
⇒ can be large even if SUSY 1L is small ! (other particles in the loops)



type:  $(\tilde{f}V\phi)$



type:  $(\tilde{f}VV)$



type:  $(fV\phi)$

### 3. Calculation of MSSM two-loop corrections

Overview:

1. Generate Feynman diagrams for  $\mu\mu\gamma$  at two-loop
2. Extract contribution of  $(g - 2)_\mu$  (given in terms of two-loop integrals)
3. Expand integrals in  $m_\mu \ll M_{\text{weak}}, m_{\tilde{f}}$
4.  $\Rightarrow$  analytic expression in  $M_{\text{weak}}, M_\phi, m_{\tilde{f}}$

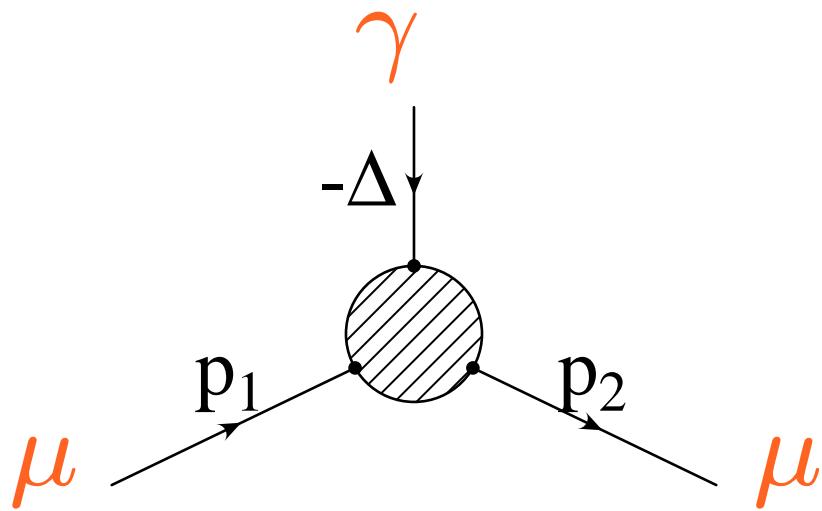
## 1. Generate Feynman diagrams for $\mu\mu\gamma$ at two-loop

- use *FeynArts* ([www.feynarts.de](http://www.feynarts.de))  
[J. Küblbeck, M. Böhm, A. Denner '90]  
[T. Hahn '00 - '03]
- use *MSSM* model file  
[T. Hahn, C. Schappacher '01]

⇒ obtain all *two-loop* diagrams  
one-loop diagrams with counter term insertion  
one-loop diagrams for renormalization

⇒ transform *diagrams to amplitudes*

## 2. Extract contribution of $(g - 2)_\mu$



Amplitude:  $M_\mu(p, \Delta)$

Expansion in  $\gamma$  momentum:

$$M_\mu(p, \Delta) \approx V_\mu(p) + \Delta^\nu T_{\nu\mu}(p)$$

( $V_\mu, T_{\nu\mu}$  given in terms  
of 2L self-energies)

next step: project out  $(g - 2)_\mu$ :

$$a \sim \frac{1}{m_\mu^2} \text{Tr} [P^\mu V_\mu + Q^\nu \mu T_{\nu\mu}]$$

Next step:

- Dirac algebra
- traces, ...
- reduction to a “basic” set of 2L integrals

⇒  $a = (g - 2)/2$  is given in terms of 2L integrals:

$$a = (g - 2)/2 = C_1 \times \text{Integral}_1 + C_2 \times \text{Integral}_2 + \dots$$

Integrals have non-zero external momentum  $p^2 = m_\mu^2$

multiple propagators

complicated numerator

masses: 0,  $m_\mu$ ,  $M_W, \dots, M_{\text{SUSY}}$

⇒ no analytic expressions available

### 3. Expand integrals in $m_\mu \ll M_{\text{weak}}, m_{\tilde{f}}$

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Leading term in  $a_\mu$ :  $\sim \frac{m_\mu^2}{M_{W,\text{SUSY}}^2} \approx 10^{-6}$

$\Rightarrow \frac{m_\mu}{M_{W,\text{SUSY}}}$  is a good expansion parameter

$\Rightarrow$  perform Taylor expansion or “Large mass expansion” of the integrals

$\Rightarrow$  all integrals are reduced to vacuum integrals:  $T_{134}, A_0, B_0$

- Further simplification of coefficients
- Insertion of analytical expressions for  $T_{134}, A_0, B_0$

$\Rightarrow$  Analytical result for  $a = (g - 2)_\mu$

$\Rightarrow$  numerical evaluation possible

(implementation in *FeynHiggs2.1*)

## Performed checks of our result:

- Cancellation of UV divergences
- Cancellation of terms  $\sim m_\mu^{-4}, m_\mu^{-2}, m_\mu^0$
- Cancellation of field renormalization constants
- Reevaluation of SM 2L diagrams from [*Czarnecki, Krause, Marciano '98*]  
⇒ perfect analytical agreement (after going to the appropriate limit)
- Going to the limit of [*A. Arhrib, S. Baek '01*] ⇒ agreement  
(i.e. [*C. Chen, C. Geng '01*] is too large by a factor of 4)

## 4. MSSM two-loop results

Our two questions:

**Q1:** How large are the complete contributions?

**Q2:** What happens if experimental constraints are taken into account?

⇒ Scan over the MSSM parameter space

$$-3 \text{ TeV} \leq \mu \leq 3 \text{ TeV}$$

$$-3 \text{ TeV} \leq A_{t,b} \leq 3 \text{ TeV}$$

$$150 \text{ GeV} \leq M_A \leq 1 \text{ TeV}$$

$$0 \leq M_{\text{SUSY}} \leq 1 \text{ TeV}$$

$$\tan \beta = 50$$

$$M_{\text{SUSY}} = M_Q = M_L = M_U = M_D = M_E \text{ (later relaxed ... )}$$

$$A_\tau = A_b$$

## Experimental constraints:

Quantity	$M_h$	$\Delta\rho^{\text{SUSY}}$	$\text{BR}(B_s \rightarrow \mu^+ \mu^-)$	$\Delta_{B \rightarrow X_s \gamma}$
strong bound	$> 111.4 \text{ GeV}$	$< 3 \times 10^{-3}$	$< 0.97 \times 10^{-6}$	$< 1.0 \times 10^{-4}$
weak bound	$> 106.4 \text{ GeV}$	$< 4 \times 10^{-3}$	$< 1.2 \times 10^{-6}$	$< 1.5 \times 10^{-4}$

$M_h$ : strong: exp. bound - 3 GeV theory uncertainty

weak: effect of  $\delta m_t^{\text{exp}} = +5 \text{ GeV}$

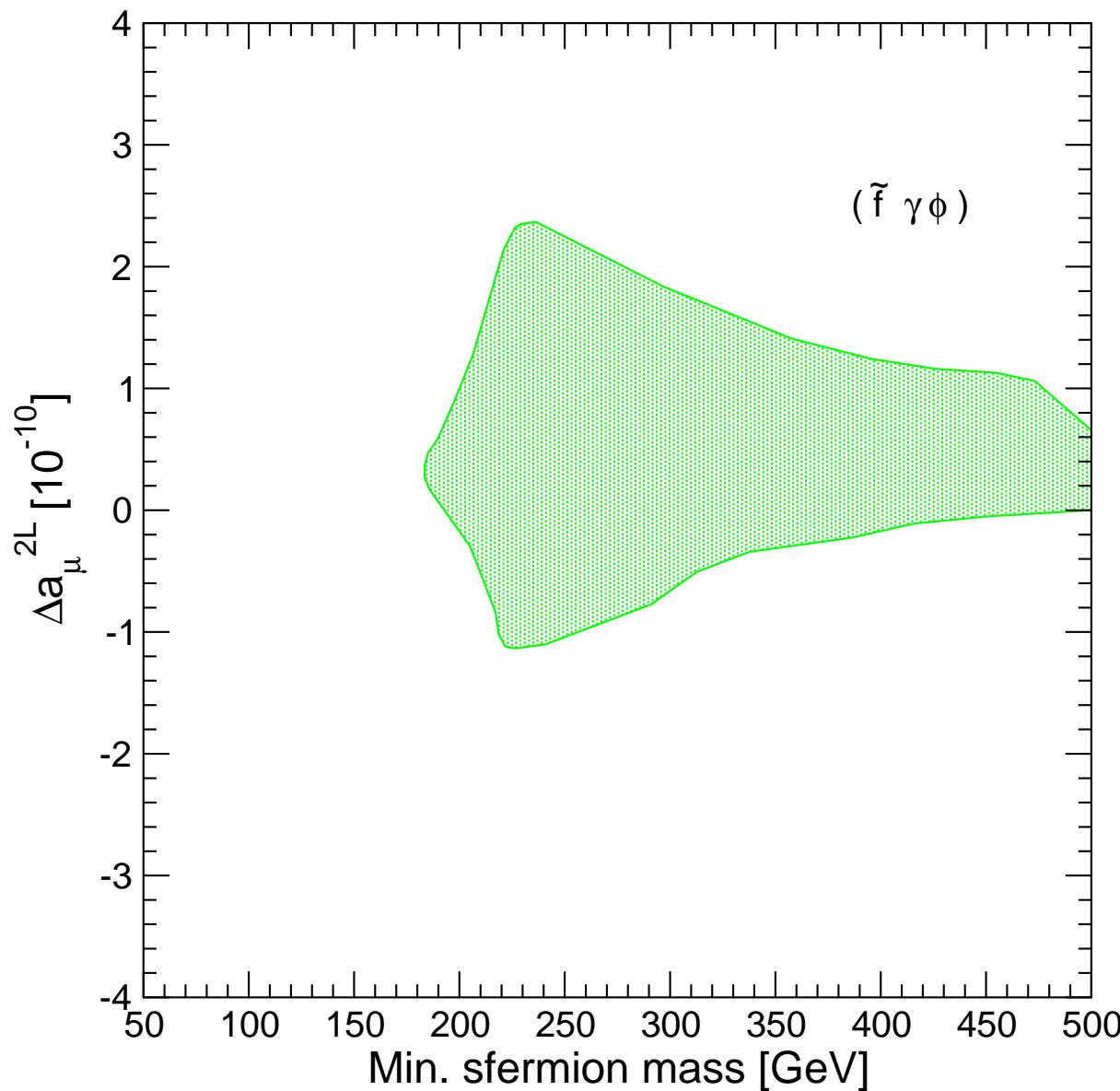
$\Delta\rho^{\text{SUSY}}$  : strong:  $2\sigma$ , weak:  $3\sigma$

$\text{BR}(B_s \rightarrow \mu^+ \mu^-)$  : strong: 90% CL, weak: 95% CL

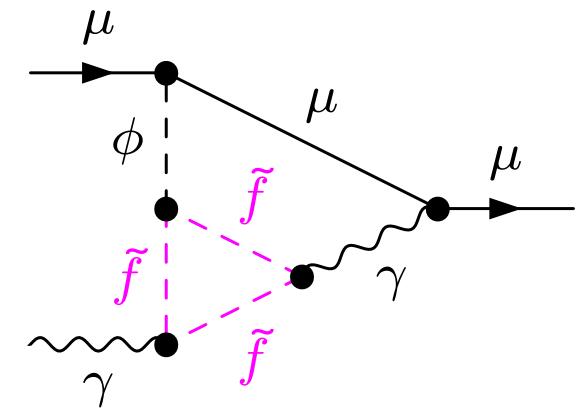
$\Delta_{B \rightarrow X_s \gamma} = |\text{BR}(B \rightarrow X_s \gamma) - 3.34 \times 10^{-4}|$

strong: 90% CL, weak: 95% CL

## Numerical results with strong bounds (I)

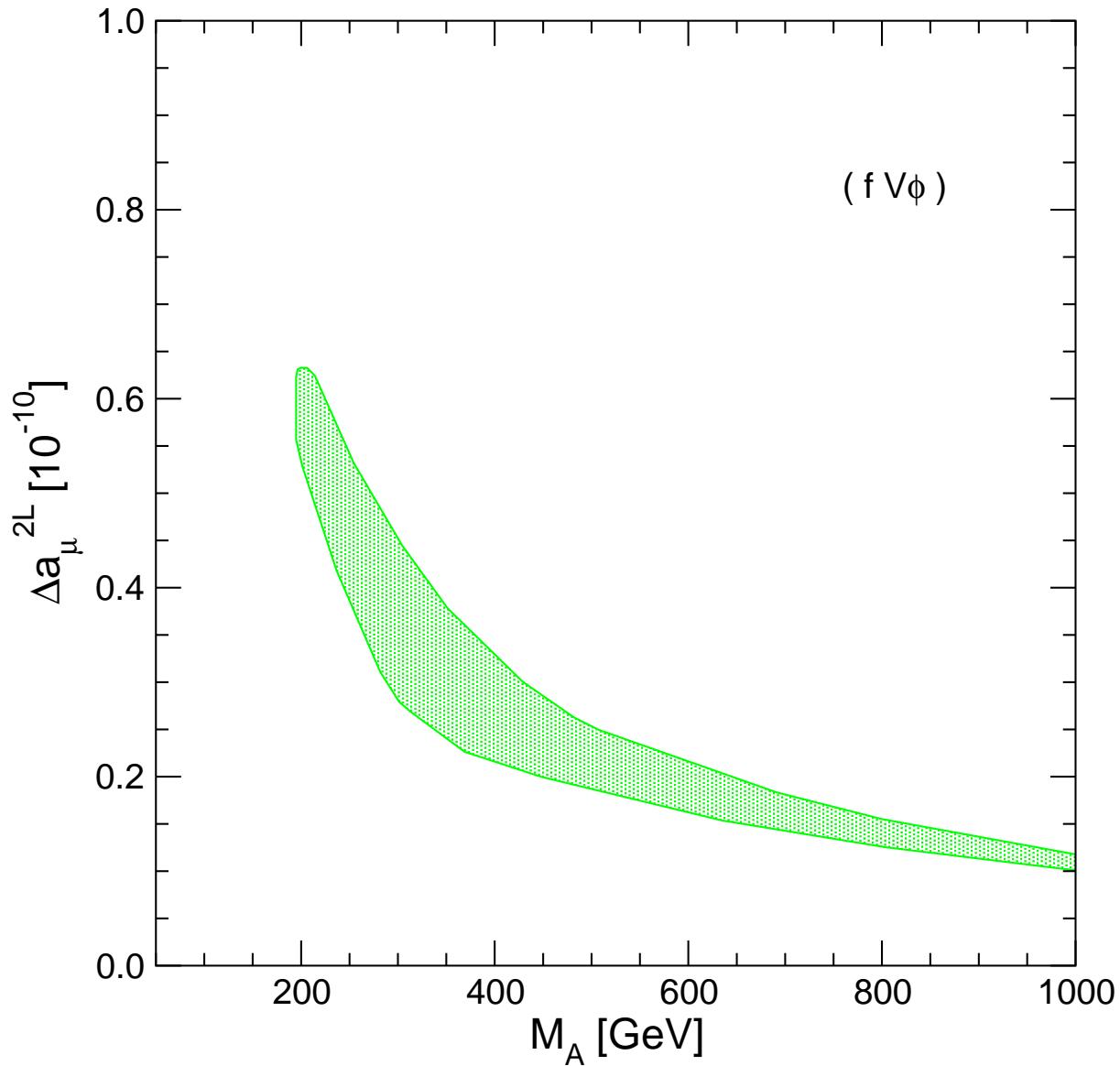


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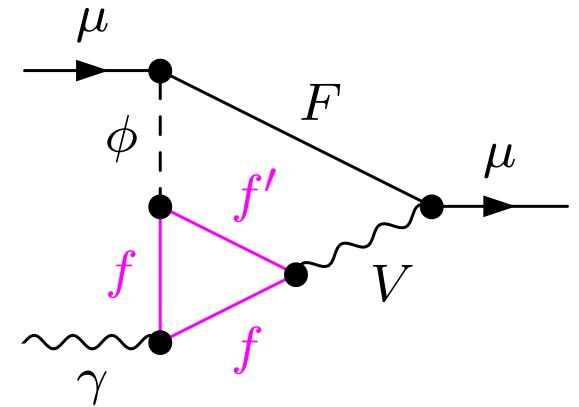


most important  
depending on  $m_{\tilde{f}}$ ,  $\mu$ ,  $A_f$ ,  
 $\tan \beta$   
significant fraction of  
current experimental error  
(Min. sferm. mass =  
 $\min\{m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}\}$ )

## Numerical results with strong bounds (II)



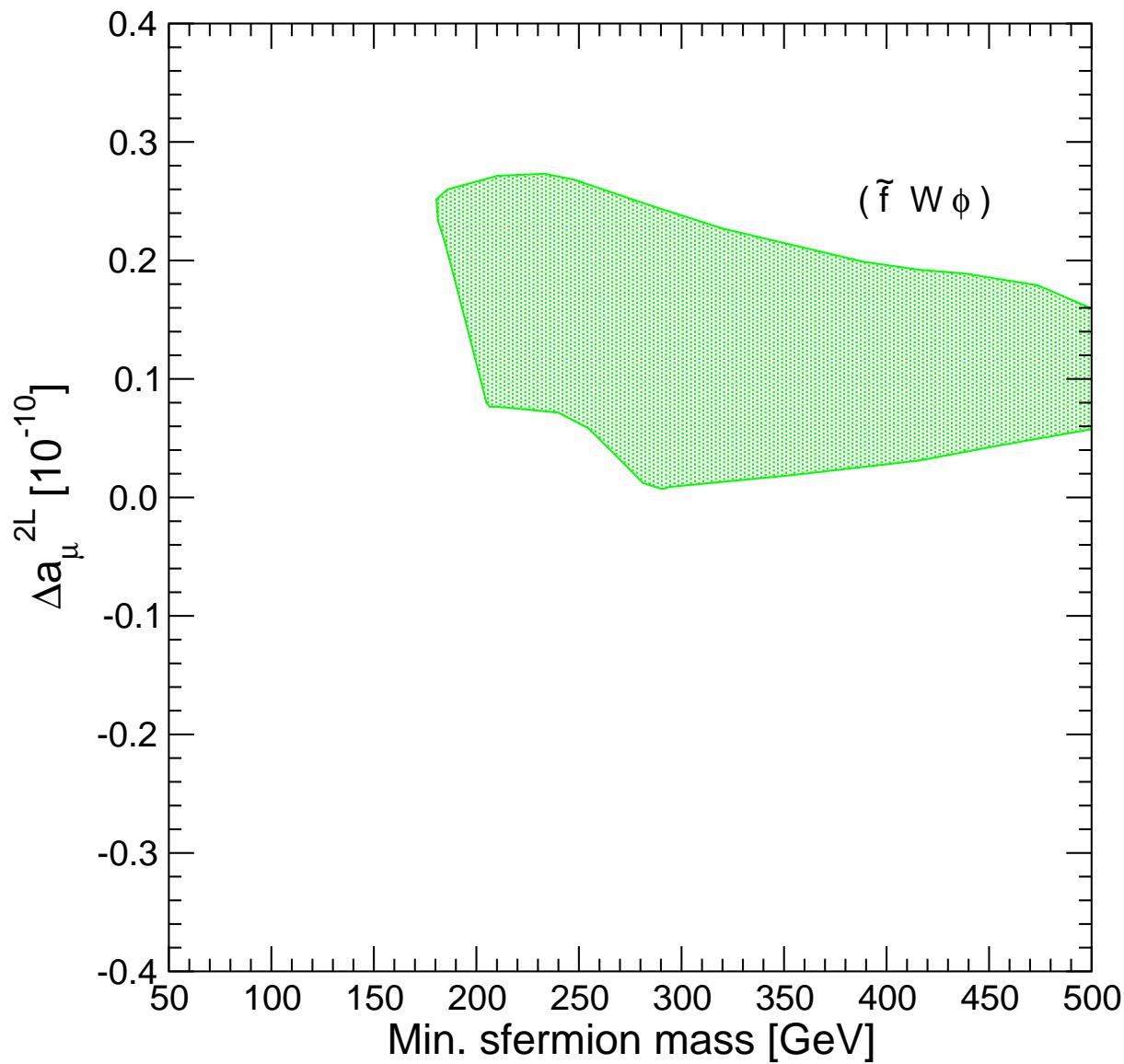
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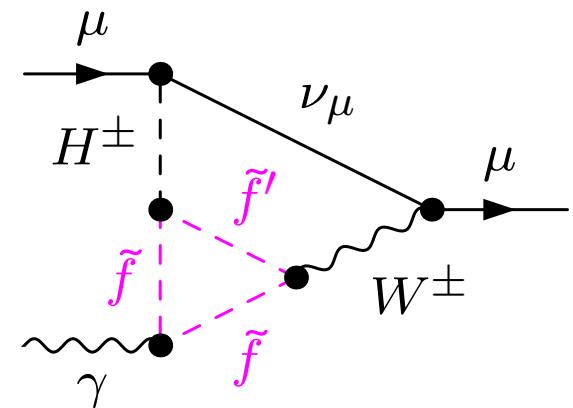
non-negligible  
depending on  $M_A \dots$

(shown here:  
MSSM - SM contribution)

## Numerical results with strong bounds (III)



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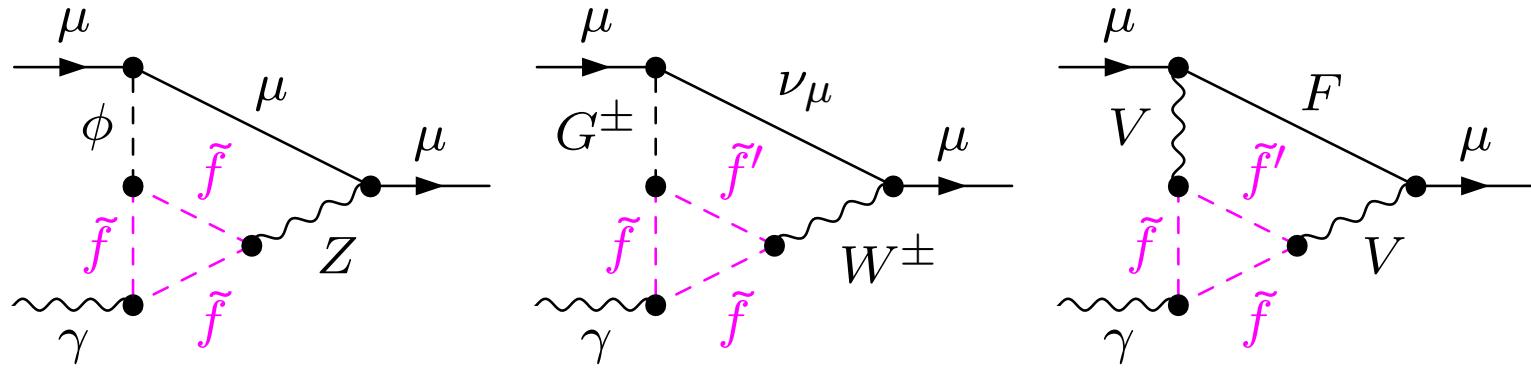


small  
depending on  $m_{\tilde{f}}$ ,  $\mu$ ,  $A_f$ ,  
 $\tan \beta$

(Min. sferm. mass =  
 $\min\{m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}\}$ )

## Numerical results with strong bounds (IV)

Other contributions:  $\lesssim 0.1 \times 10^{-10}$



$((\tilde{f}VV))$  also includes  $\Delta r$  contribution from reparameterization of one-loop result)

⇒ concentrate on  $(\tilde{f}\gamma\phi)$ ,  $(\tilde{f}W\phi)$  for further investigations

$\Delta a_\mu^{2L} \lesssim 3 \times 10^{-10}$  for strong constraints

## Q2: What are the effects of the experimental constraints?

⇒ apply weak constraints ⇒ apply strong constraints

### A2: Effects of weak experimental constraints:

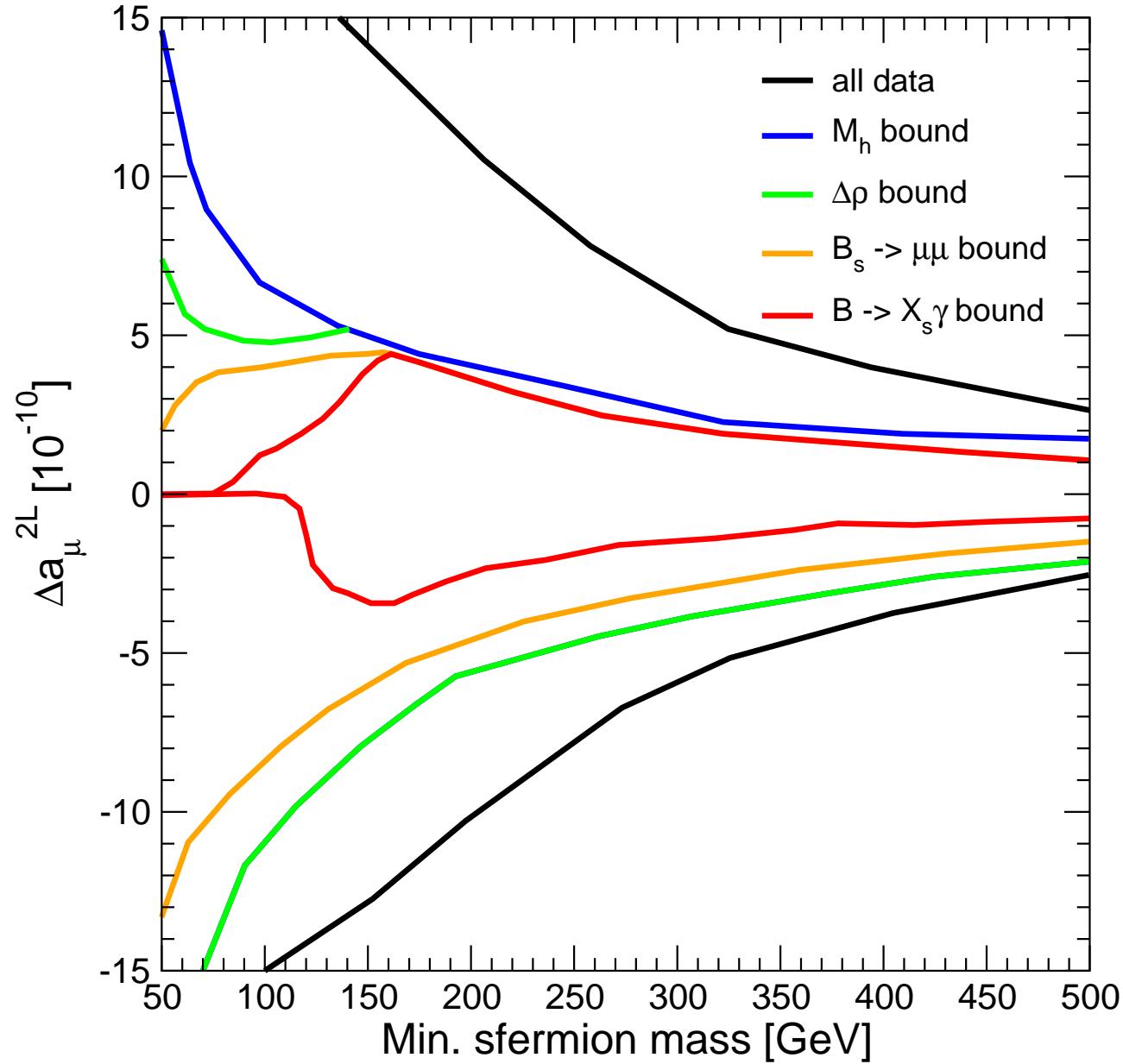
- take the whole data sample
- apply more and more **weak** experimental constraints

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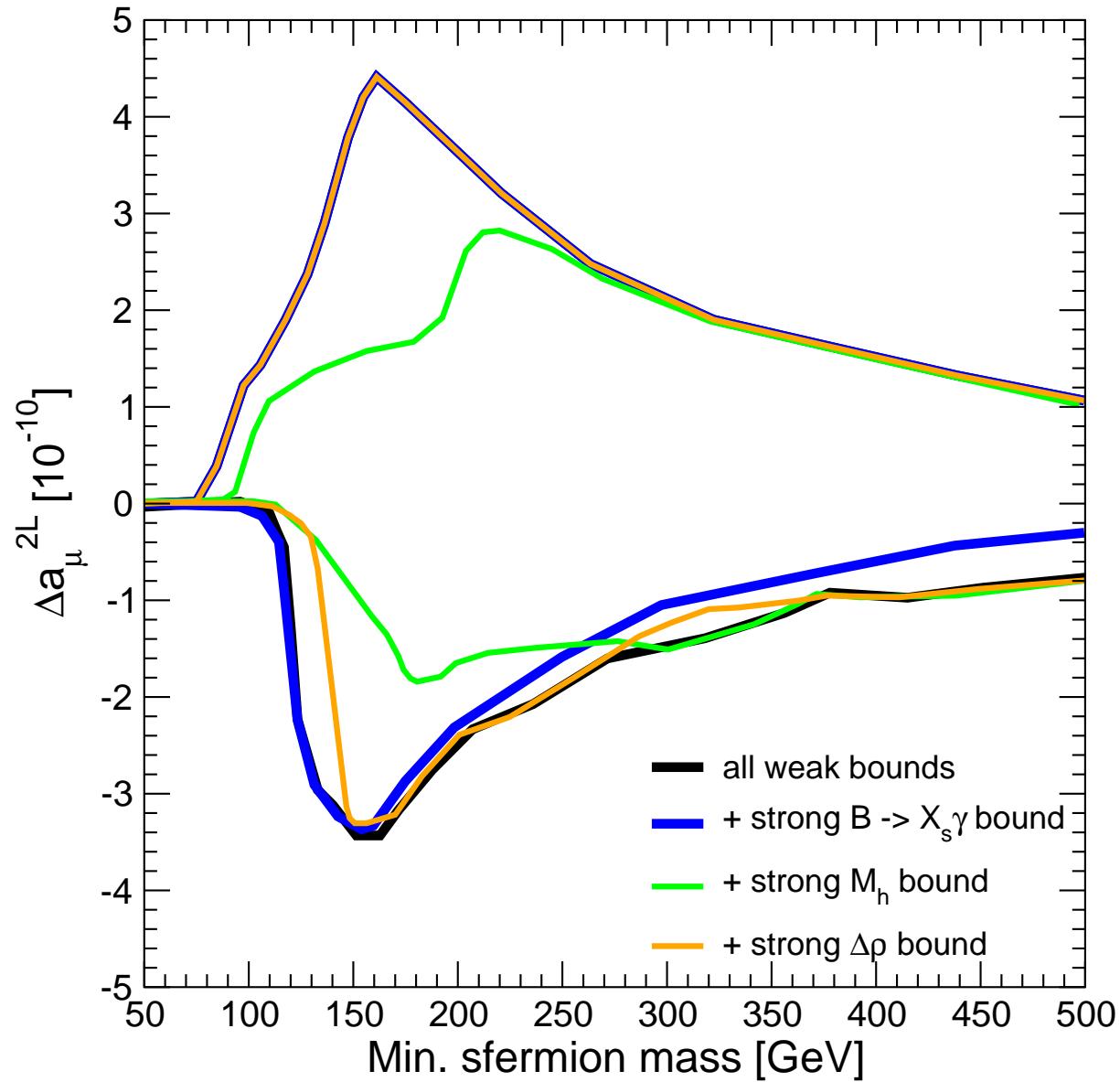
### Observations:

- no constraints ⇒ corrections up to  $20 \times 10^{-10}$  possible
- $M_h$  bound very strong,  $|\Delta a_\mu^{2L}| \lesssim 5 \times 10^{-10}$  for  $m_{\tilde{f}} \gtrsim 150$  GeV
- $\text{BR}(B \rightarrow X_s \gamma)$  bound cuts away everything for  $m_{\tilde{f}} \lesssim 150$  GeV
- weak bounds ⇒  $|\Delta a_\mu^{2L}| \lesssim 5 \times 10^{-10}$

## Application of weak constraints:



## A2': Effects of strong experimental constraints:



Most effective:  $M_h$  bound

⇒ reduction to

$$|\Delta a_\mu^{2L}| \lesssim 3 \times 10^{-10}$$

(up to  $1/2\sigma$  of current experimental precision)

## **A2": How to "avoid" the experimental constraints:**

Main reason for "small" results: experimental bounds constrain  $|\mu| \lesssim 1$  TeV

However:

more freedom for  $\mu$  with non-universal soft SUSY-breaking parameters

Example (investigated here):

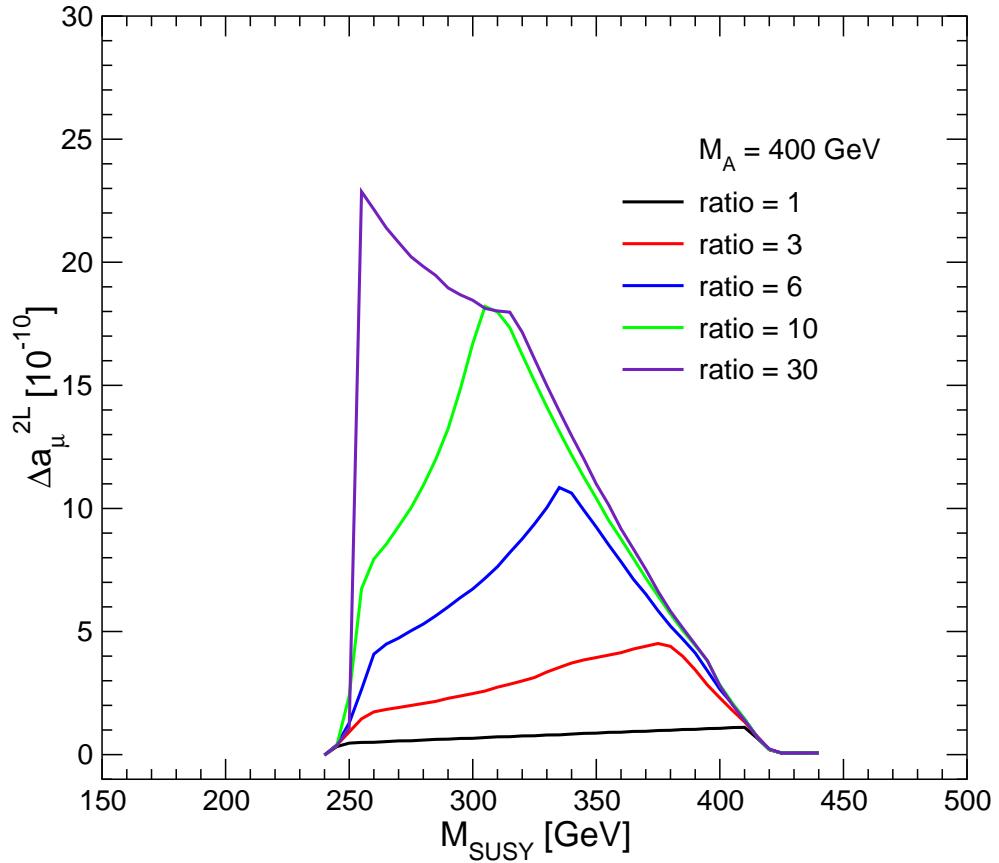
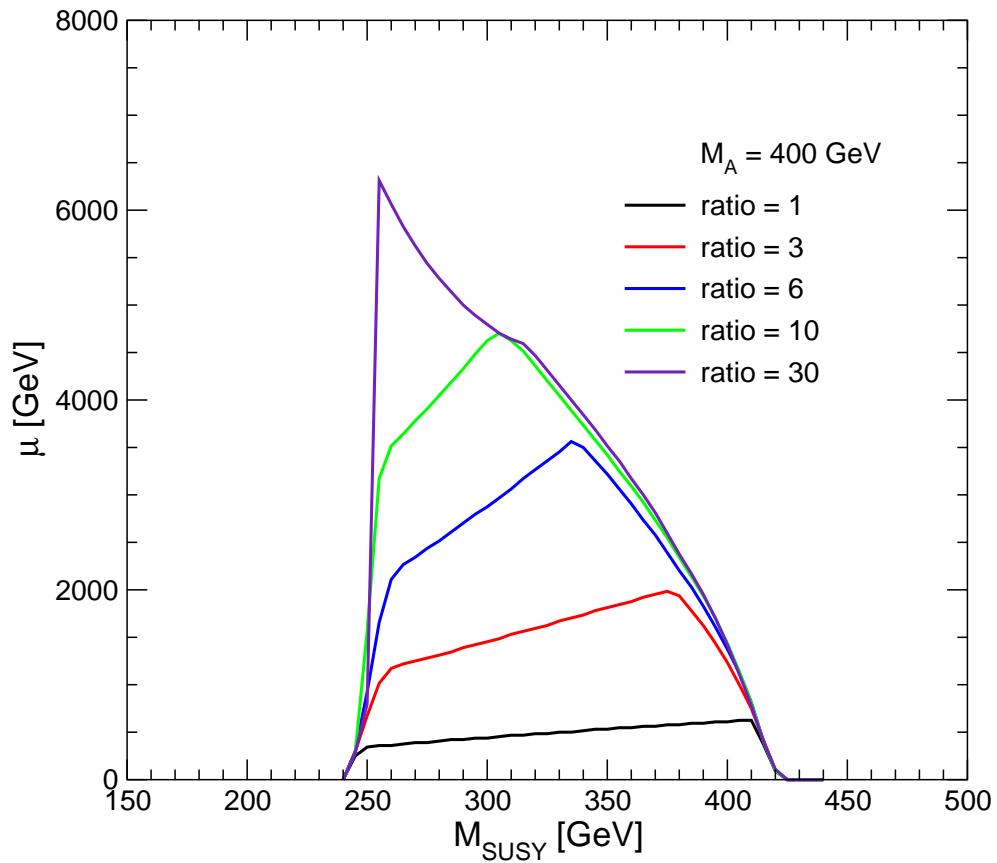
$$M_{\text{SUSY}} = M_Q = M_U = M_L \neq M_D = M_E$$

(disconnects  $\tilde{t}$  and  $\tilde{b}$  sector)

To obtain "extreme" results:

$$M_A = 400 \text{ GeV}, m_{\tilde{t}_1} = 150 \text{ GeV}, \text{ratio} := M_D/M_U \neq 1$$

## Results for relaxed universality condition:



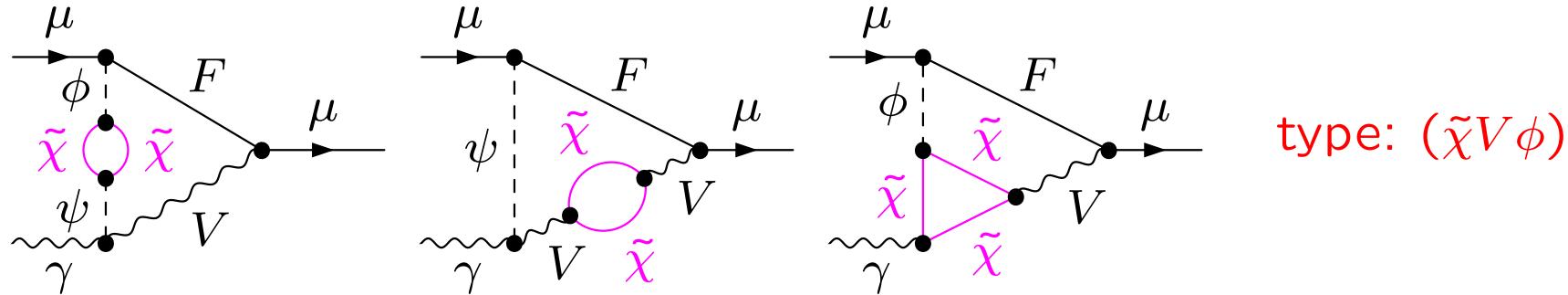
$M_D/M_U \gg 1 \Rightarrow$  very large  $\mu$  possible

$\Rightarrow \Delta a_\mu^{2L} > 20 \times 10^{-10}$  possible

(this shows how “difficult” it is to obtain large corrections)

## Very recent results

All diagrams with a **closed chargino/neutralino loop**:



Approximation formula:

$$\Delta a_{\mu}^{2L\tilde{\chi}} \approx 0.27 \times 10^{-10} \times \left( \frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 \times \tan \beta \times \text{sgn}(\mu)$$

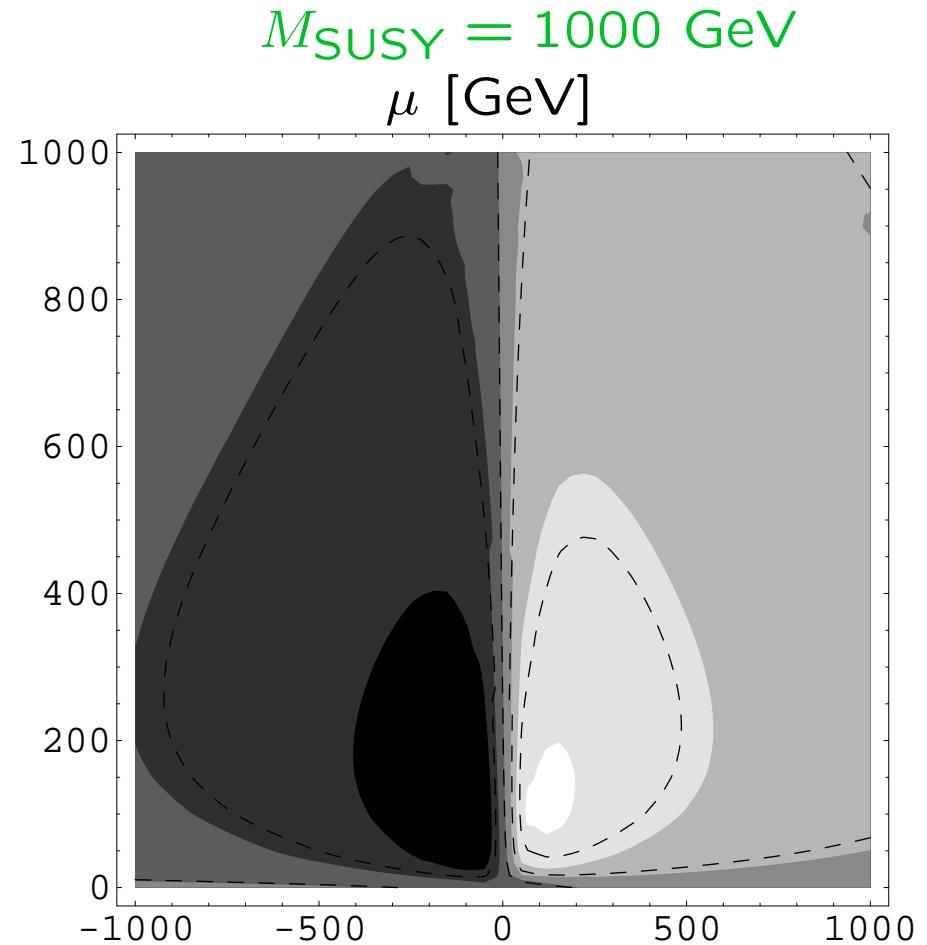
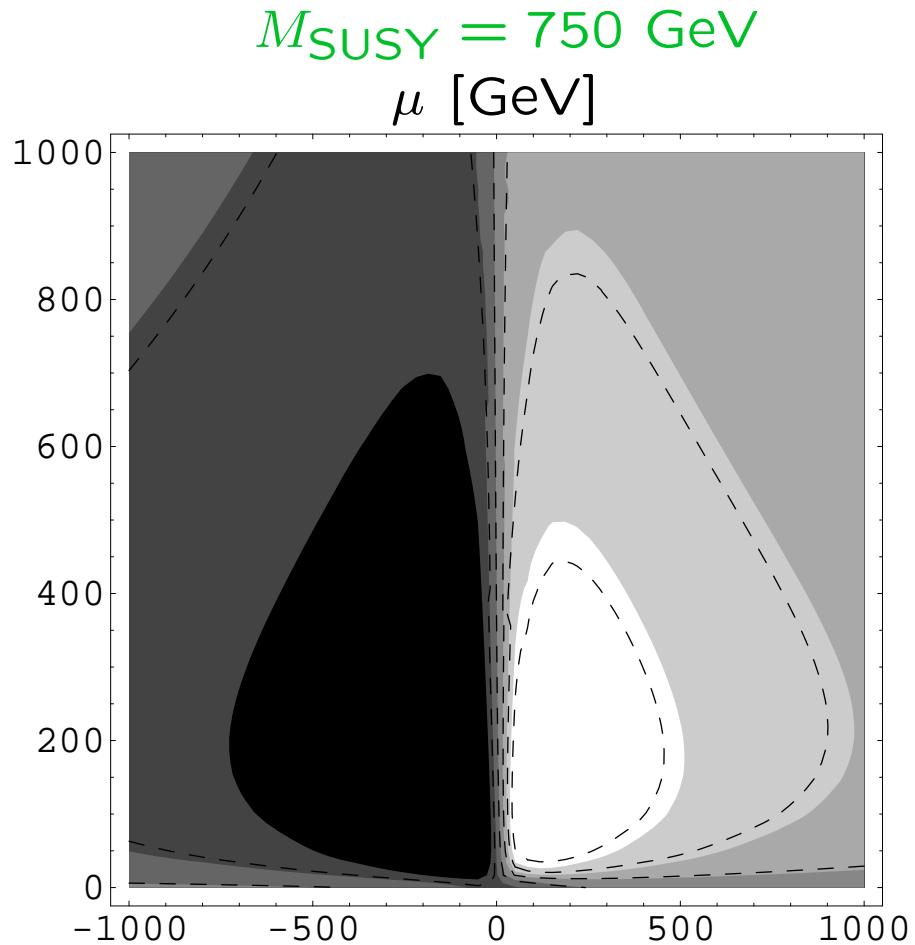
Same particles as at one-loop  $\Rightarrow$  add one- and two-loop corrections

Parameter:

$$M_{\text{SUSY}}(\text{3rd gen.}) = 1000 \text{ GeV}, M_A = 200 \text{ GeV}, \tan \beta = 50$$

$\rightarrow T$

$\mu$ - $M_2$  plane: indicated: 0, 1, 2, 3, ...  $-\sigma$  areas:



dashed: one-loop only

colored areas: one+two-loop  $\Rightarrow$  non-negligible effect

## 5. Conclusions

- Precision observables can give valuable information about the “true” Lagrangian
- new experimental result for  $a_\mu$ :  
 $a_\mu^{\text{exp}} - a_\mu^{\text{theo}} \approx (29 \pm 10) \times 10^{-10}$  :  $2.5 - 3.4\sigma$  ( $e^+e^-$  data, no MV)
- SUSY could easily explain “discrepancy”  
 $a_\mu$  can provide bounds on SUSY parameters
- SUSY enhancement factors:  $m_\mu \tan \beta \times \mu m_t$ ,  $A_b m_b \tan \beta$
- Evaluation of two-loop contributions with closed  $f/\tilde{f}$  loop
- Expansion  $\Rightarrow$  analytic result in vacuum integrals
- strong exp. bounds  $\Rightarrow |\Delta a_\mu^{2L}| \lesssim 3 \times 10^{-10}$  ( $\sim 1/2\sigma$  of exp. error)
- weak exp. bounds  $\Rightarrow |\Delta a_\mu^{2L}| \lesssim 5 \times 10^{-10}$
- breaking of universality of SUSY-breaking terms  $\Rightarrow \Delta a_\mu^{2L} \lesssim 20 \times 10^{-10}$

**Outlook:** evaluation of further two-loop corrections